Exercise 50

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = x^3 - 6x^2 + 5, \quad [-3, 5]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^3 - 6x^2 + 5) \\ &= (3x^2) - 6(2x) + 5(0) \\ &= 3x^2 - 12x \end{aligned}$$

Set $f'(x) = 0$ and solve for x.

$$\begin{aligned} 3x^2 - 12x &= 0 \\ 3x(x - 4) &= 0 \\ x &= \{0, 4\} \end{aligned}$$

$x = 0$ and $x = 4$ are within $[-3, 5]$, so evaluate f at these values.

$$f(0) = (0)^3 - 6(0)^2 + 5 = 5 \qquad \text{(absolute maximum)}$$

$$f(4) = (4)^3 - 6(4)^2 + 5 = -27$$

Now evaluate the function at the endpoints of the interval.

$$f(-3) = (-3)^3 - 6(-3)^2 + 5 = -76 \qquad \text{(absolute minimum)}$$

$$f(5) = (5)^3 - 6(5)^2 + 5 = -20$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-3, 5]$.

The graph of the function below illustrates these results.